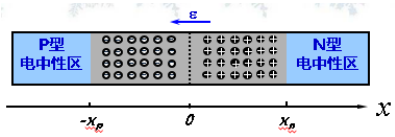
四：P-N结的自建电势与自建电场

Thursday, March 5, 2020 1:13 PM

1. Possion方程：



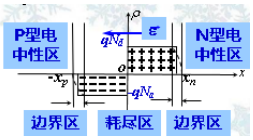
$$\frac{d^2\psi}{dx^2}=-\frac{d\vec{\varepsilon}}{dx}=-\frac{\rho}{\varepsilon_0\varepsilon_{rs}}=-\frac{q(p-n+N_d-N_a)}{\varepsilon_s}$$

内建电势差：

$$\psi_0=\psi_n-\psi_p=V_T\ln\frac{n_0p_0}{n_i^2}=V_T\ln\frac{N_dN_a}{n_i^2}$$

2. 突变结的电场和电势分布：

已知电荷分布，求电场和电势分布问题。



Possion方程：

$$\frac{d^2\psi}{dx^2}=-\frac{d\vec{\varepsilon}}{dx}=-\frac{\rho}{\varepsilon_0\varepsilon_{rs}}=-\frac{q(p-n+N_d-N_a)}{\varepsilon_s}$$

a. 电场分布：

$$\rho(x)=\begin{cases}0,(x<-x_p,x>x_n)\text{电中性区}\\-qN_a,(-x_p<x<0)\text{耗尽区}\\qN_d,(0<x<x_n)\text{耗尽区}\end{cases}$$

边界层的宽度约为德拜屏蔽长度L_p的3倍，一般小于$0.1\mu m$，可以忽略；

i. 电中性区：

$$\vec{\varepsilon}(x)=0$$

电中性区无电荷，没有电场线，所有电场线都集中于耗尽区；

ii. P区耗尽层$(-x_p<x<0)$：

Possion方程：

$$\frac{d\vec{\varepsilon}}{dx}=-\frac{qN_a}{\varepsilon_s}$$

$$\Rightarrow\vec{\varepsilon}(x)=-\int\frac{qN_a}{\varepsilon_s}dx=-\frac{qN_a}{\varepsilon_s}x+C_p$$

电场强度连续性边界条件：

$$\vec{\varepsilon}(x)\Big|_{x=-x_p}=0$$

$$\Rightarrow\vec{\varepsilon}(-x_p)=\frac{qN_a}{\varepsilon_s}x_p+C_p=0$$

$$\Rightarrow C_p=-\frac{qN_a}{\varepsilon_s}x_p$$

$$\vec{\varepsilon}(x)=-\frac{qN_a}{\varepsilon_s}(x-x_p),(-x_p<x<0)$$

iii. N区耗尽层$(0<x<x_n)$：

Possing方程：

$$\frac{d\vec{\varepsilon}}{dx}=\frac{qN_d}{\varepsilon_s}\text{(电离施主带正电荷)}$$

$$\vec{\varepsilon}(x)=\int\frac{qN_d}{\varepsilon_s}dx=\frac{qN_d}{\varepsilon_s}x+C_n$$

$$\Rightarrow \varepsilon(x) = \int \frac{\quad}{\varepsilon_s} dx = \frac{\quad}{\varepsilon_s} x + C_n$$

电场强度连续性边界条件：

$$\vec{\varepsilon}(x)\Big|_{x=x_n} = 0$$

$$\Rightarrow \vec{\varepsilon}(x_n) = \frac{qN_d}{\varepsilon_s} x_n + C_n$$

$$\Rightarrow C_n = -\frac{qN_d}{\varepsilon_s} x_n$$

$$\vec{\varepsilon}(x) = -\frac{qN_d}{\varepsilon_s}(x_n - x), (x_n > x > 0)$$

iv. 讨论：

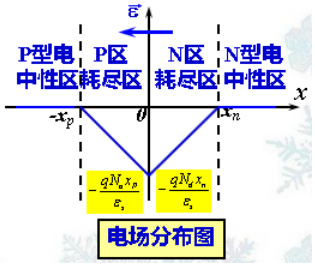
$x = 0$处电场连续，且最强：

$$\overrightarrow{\varepsilon_m} = -\frac{qN_d}{\varepsilon_s} x_n = -\frac{qN_a}{\varepsilon_s} x_p$$

$$\Rightarrow N_d x_n = N_a x_p$$

1) P区耗尽层中单位面积内的负电荷数=N区耗尽层中单位面接内的正电荷数；
2) P区耗尽层中的负电荷数=N区耗尽层中正电荷数；
3) 掺杂浓度越高的一侧，空间电荷区越薄；
4) 最大电场强度位于P、N区的交界面处；

v. 电场分布图：



b. 电势分布：

i. 电中性区$(-x_p \geq x, x \leq x_n)$：

$$\vec{\varepsilon}(x) = -\frac{d\psi(x)}{dx} = 0$$

$$\psi(x) = C$$

取P型中性区的电势为零电势，则：

$$\psi(x) = \begin{cases} 0, (x < -x_p) \\ \psi_0, (x > x_n) \end{cases}, \psi_0 = V_T \ln \frac{N_a N_d}{n_i^2}$$

内建电势差完全降落在耗尽区（空间电荷区），电中性区没有压降；

ii. P取耗尽层$(-x_p \leq x \leq 0)$：

$$-\frac{d\psi(x)}{dx} = \vec{\varepsilon}(x) = -\frac{qN_a}{\varepsilon_s}(x + x_p)$$

$$\Rightarrow \psi(x) = -\int \vec{\varepsilon}(x) dx = \int \frac{qN_a}{\varepsilon_s}(x + x_p) dx$$

$$\Rightarrow \psi(x) = \frac{qN_a}{2\varepsilon_s}(x + x_p)^2 + C_p$$

有边界条件：

$$\psi(x)\Big|_{x=-x_p} = 0$$

$$\Rightarrow C_p = 0$$

$$\psi(x) = \frac{qN_a}{2\varepsilon_s}(x + x_p)^2$$

iii. N区耗尽层$(x_n \geq x \geq 0)$：

$$-\frac{d\psi(x)}{dx} = \vec{\varepsilon}(x) = -\frac{qN_d}{\varepsilon_s}(x_n - x)$$

$$\Rightarrow \psi(x) = -\int \vec{\varepsilon}(x)dx = \int \frac{qN_d}{\varepsilon_s}(x_n - x)dx$$

$$\Rightarrow \psi(x) = -\frac{qN_d}{2\varepsilon_s}(x_n - x)^2 + C_n$$

有边界条件：

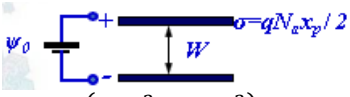
$$\psi(x)\Big|_{x=0} = \frac{qN_a}{2\varepsilon_s}x_p^2$$

$$\Rightarrow C_n = \frac{q\left(N_a x_p^2 + N_d x_n^2\right)}{2\varepsilon_s}$$

$$\psi(x) = -\frac{qN_d}{2\varepsilon_s}(x_n - x)^2 + \frac{q\left(N_a x_p^2 + N_d x_n^2\right)}{2\varepsilon_s}$$

$$= -\frac{qN_d}{2\varepsilon_s}(x^2 - 2x_n x) + \frac{qN_a x_p^2}{2\varepsilon_s}$$

iv. 讨论：电势在$x = x_n$处连续



$$\psi_0 = \frac{q\left(N_a x_p^2 + N_d x_n^2\right)}{2\varepsilon_s} = V_T \ln\frac{N_a N_d}{n_i^2}$$

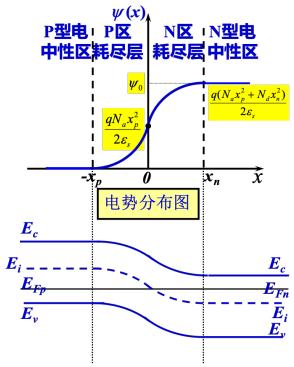
$$= \frac{qN_a x_p\left(x_n + x_p\right)}{2\varepsilon_s} = -\frac{1}{2}\varepsilon_m W$$

$$\left(W = \sqrt{\frac{2\varepsilon_s\psi_0}{q}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)}\right)$$

$$\Rightarrow \vec{s} = \frac{\psi_0}{W} = -\frac{\varepsilon_m}{2} = \frac{\sigma}{\varepsilon_s} = \frac{qN_a x_p}{2\varepsilon_s}$$

对于均匀掺杂的半导体P-N结，电势分布为空间坐标的二次函数，因此电势能的变化也是空间坐标的二次函数；

v. 电势分布图：



c. 空间电荷区宽度：

$$N_d x_n = N_a x_p$$

$$\Rightarrow x_p = \frac{N_d x_n}{N_a}$$

$$\psi_0 = \frac{q\left(N_a x_p^2 + N_d x_n^2\right)}{2\varepsilon_s}$$

$$\Rightarrow N_a x_p^2 + N_d x_n^2 = \frac{2\varepsilon_s\psi_0}{q}$$

$$\Rightarrow \frac{N_d(N_a + N_d)}{N_a}x_n^2 = \frac{2\varepsilon_s\psi_0}{q}$$

$$\Rightarrow x_n = \sqrt{\frac{2\varepsilon_s\psi_0}{q}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a + N_d}\right)}$$

同理得出x_p，结果为：

$$\left\{x_n = \sqrt{\frac{2\varepsilon_s\psi_0}{q}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a + N_d}\right)}\right.$$

$$\begin{cases} x_n = \sqrt{\dfrac{q}{2\varepsilon_s\psi_0}\left(\dfrac{N_d}{N_a}\right)\left(\dfrac{1}{N_a+N_d}\right)} \\ x_p = \sqrt{\dfrac{2\varepsilon_s\psi_0}{q}\left(\dfrac{N_d}{N_a}\right)\left(\dfrac{1}{N_a+N_d}\right)} \end{cases}$$

$$W = x_n + x_p = \left(1+\frac{N_d}{N_a}\right)x_n$$

$$\Rightarrow W = \sqrt{\frac{2\varepsilon_s\psi_0}{q}\left(\frac{N_a+N_d}{N_aN_d}\right)}$$

d. 例：

i. 已知：$Si\ P-N$结，$T=300K, N_a=10^{16}cm^{-3}, N_d=10^{15}cm^{-3}, n_i=1.5\times10^{10}cm^{-3}$。计算P-N结得空间电荷区宽度及内建电场。

解：

计算内建电势差(P-N结势垒)：

$$\psi_0 = V_T \ln\frac{N_dN_a}{n_i^2} = 0.026eV \ln\frac{10^{16}cm^{-3}\times10^{15}cm^{-3}}{(1.5\times10^{10}cm^{-3})^2}$$
$$\approx 0.637V$$

计算空间电荷区宽度：

$$\varepsilon_{si} = 11.8\varepsilon_0$$
$$\varepsilon_0 = 8.854187817\times10^{-12}C^2/(\mathrm{N}\cdot\mathrm{m}^2)$$
$$W = \sqrt{\frac{2\varepsilon_s\psi_0}{q}\left(\frac{N_a+N_d}{N_aN_d}\right)}$$
$$= \sqrt{\frac{2\times11.8\times8.85\times10^{-12}C^2/(N\cdot m^2)\times0.637V}{1.6\times10^{-19}C}\times\frac{10^{16}cm^{-3}+10^{15}cm^{-3}}{(1.5\times10^{10}cm^{-3})^2}}$$
$$\approx 9.5\times10^{-7}m$$

$$x_p = \frac{N_dx_n}{N_a} = 0.1x_n$$

$$\Rightarrow \begin{cases} x_n = \dfrac{N_aW}{N_a+N_d} = \dfrac{10W}{11} \approx 0.869\mu m \\ x_p = \dfrac{N_dW}{N_a+N_d} = \dfrac{W}{11} \approx 0.0869\mu m \end{cases}$$，空间电荷区主要位于低掺杂一侧；

$$\overrightarrow{\varepsilon_m} = -\frac{qN_dx_n}{\varepsilon_s} = -\frac{1.6\times10^{-19}C\times10^{15}cm^{-3}\times8.69\times10^{-5}m}{11.8\times8.85\times10^{-12}C^2/(N\cdot m^2)}$$
$$\approx 1.33\times10^6\ V/m$$

$$\vec{\varepsilon} = \frac{\psi_0}{W} = \frac{0.637V}{9.56\times10^{-7}m} \approx 6.66\times10^5 V/m$$

P-N结势垒虽然很小，但是结区得内建电场却很强，因为结区非常薄；

ii. 已知：$Si\ P-N$结，$T=300K, N_a=5\times10^{16}cm^{-3}, N_d=5\times10^{15}cm^{-3}$，零偏压。

求：$x_n, x_p, W, \varepsilon_m$；

解：

计算内建电势差：

$$\psi_0 = V_T \ln\frac{N_aN_d}{n_i^2} = 0.026eV \ln\frac{5\times10^{16}cm^{-3}\times5\times10^{15}cm^{-3}}{(1.5\times10^{10}cm^{-3})^2}$$
$$\approx 0.72V$$

计算空间电荷区宽度：

$$W = \sqrt{\frac{2\varepsilon_s\psi_0}{q}\left(\frac{N_a+N_d}{N_aN_d}\right)}$$
$$= \sqrt{\frac{2\times11.8\times8.85\times10^{-12}C^2/(N\cdot m^2)\times0.72V}{1.6\times10^{-19}C}\times\frac{5\times10^{16}cm^{-3}+5\times10^{15}cm^{-3}}{5\times10^{16}cm^{-3}\times5\times10^{15}cm^{-3}}}$$
$$\approx 4.55\times10^{-7}m$$

$$x_p = \frac{N_dx_n}{N_a} = 0.1x_n$$

$$\Rightarrow \begin{cases} x_n = \dfrac{N_a W}{N_a + N_d} = \dfrac{10W}{11} = \dfrac{10\times 4.55\times 10^{-7} m}{11} \approx 4.14\times 10^{-7} m \\ x_p = \dfrac{N_d W}{N_a + N_d} = \dfrac{W}{11} = \dfrac{4.55\times 10^{-7} m}{11} \approx 4.14\times 10^{-8} m \end{cases}$$

$$\overrightarrow{\varepsilon_m} = -\frac{qN_d x_n}{\varepsilon_s} = -\frac{1.6\times 10^{-19} C\times 5\times 10^{15} cm^{-3}\times 4.14\times 10^{-7} m}{11.8\times 8.85\times 10^{-12} C^2/(N\cdot m^2)}$$
$$\approx 3.17\times 10^6 V/m$$

iii. 已知：$GaAs\ P-N$结，$T=300K, N_a = 5\times 10^{16} cm^{-3}, N_d = 5\times 10^{15} cm^{-3}$,零偏压。

求：$x_n, x_p, W, \varepsilon_m$；

解：

计算内建电势差：

$$\psi_0 = V_T \ln\frac{N_a N_d}{n_i^2} = 0.026eV \ln\frac{5\times 10^{16} cm^{-3}\times 5\times 10^{15} cm^{-3}}{(1.8\times 10^6 cm^{-3})^2}$$
$$\approx 1.19V$$

计算空间电荷区宽度：

$$\varepsilon_{GaAs} = 13.18\varepsilon_0$$
$$n_{i_{GaAs}} = 1.8\times 10^6 cm^{-3}$$
$$W = \sqrt{\frac{2\varepsilon_s\psi_0}{q}\left(\frac{N_a + N_d}{N_a N_d}\right)}$$
$$= \sqrt{\frac{2\times 13.18\times 8.85\times 10^{-12} C^2/(N\cdot m^2)\times 1.19V}{1.6\times 10^{-19} C}\times\frac{5\times 10^{16} cm^{-3} + 5\times 10^{15} cm^{-3}}{5\times 10^{16} cm^{-3}\times 5\times 10^{15} cm^{-3}}}$$
$$\approx 6.18\times 10^{-7} m$$

$$x_p = \frac{N_d x_n}{N_a} = 0.1 x_n$$

$$\Rightarrow \begin{cases} x_n = \dfrac{N_a W}{N_a + N_d} = \dfrac{10W}{11} = \dfrac{10\times 6.18\times 10^{-7} m}{11} \approx 5.61\times 10^{-7} m \\ x_p = \dfrac{N_d W}{N_a + N_d} = \dfrac{W}{11} = \dfrac{6.18\times 10^{-7} m}{11} \approx 5.61\times 10^{-8} m \end{cases}$$

$$\overrightarrow{\varepsilon_m} = -\frac{qN_d x_n}{\varepsilon_s} = -\frac{1.6\times 10^{-19} C\times 5\times 10^{15} cm^{-3}\times 5.61\times 10^{-7} m}{13.18\times 8.85\times 10^{-12} C^2/(N\cdot m^2)}$$
$$\approx 3.84\times 10^6 V/m$$

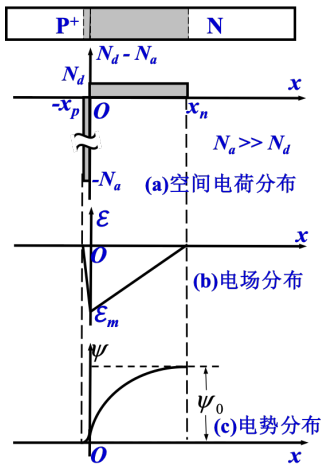
3. 单边突变结：

P-N结得一侧得掺杂浓度远远高于另一侧；

例如：结深很浅得扩散结可以近似看作单边突变结；

若，$N_a \gg N_d$称为P^+N结；若$N_d \gg N_a$，称为PN^+结；

a. P^+N结($N_a \gg N_d$)：



i. 空间电荷区宽度：

$$W \approx x_n \approx \sqrt{\frac{2\varepsilon\psi_0}{qN_d}}$$

ii. 电场分布$(x_n > x > 0)$:

$$\overrightarrow{\varepsilon_m} \approx -\frac{qN_dW}{\varepsilon_s}$$

$$\vec{\varepsilon}(x) \approx \overrightarrow{\varepsilon_m}\left(1-\frac{x}{W}\right)$$

iii. 电势分布$(x_n > x > 0)$:

$$\psi(x) \approx \begin{cases} 0, (-x_p < x < 0) \\ -\frac{qN_d}{2\varepsilon_s}(x^2 - 2xW), (x_n > x > 0) \end{cases}$$

$$\psi_0 \approx \frac{qN_dW^2}{2\varepsilon_s}$$

b. PN^+结$(N_a \ll N_d)$:

i. 空间电荷区宽度:

$$W \approx x_p = \sqrt{\frac{2\varepsilon_s\psi_0}{q}\left(\frac{N_d}{N_a}\right)\left(\frac{1}{N_a+N_d}\right)}$$

$$\approx \sqrt{\frac{2\varepsilon_s\psi_0}{qN_a}}$$

ii. 电场分布$(-x_p < x < 0)$:

$$\overrightarrow{\varepsilon_m} \approx -\frac{qN_a}{\varepsilon_s}W$$

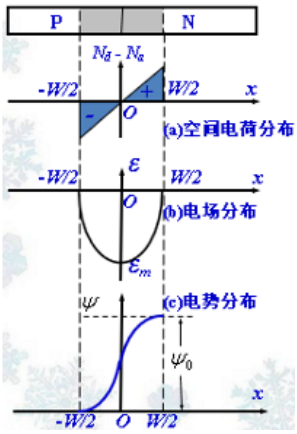
$$\vec{\varepsilon}(x) = \overrightarrow{\varepsilon_m}(x_n - x) \approx \overrightarrow{\varepsilon_m}\left(1-\frac{x}{W}\right)$$

iii. 电势分布$(x_n > x > 0)$:

$$\psi(x) \approx \begin{cases} -\frac{qN_d}{2\varepsilon_s}(x^2 - 2xW), (-x_p < x < 0) \\ 0, (0 < x < x_n) \end{cases}$$

$$\psi_0 \approx \frac{qN_aW^2}{2\varepsilon_s}$$

4. 线性缓变结



a. 空间电荷分布:

$$N_d - N_A = ax, \left(\frac{W}{2} > x > -\frac{W}{2}\right)$$

b. 电场分布:

Possion方程:

$$\frac{d\vec{\varepsilon}}{dx} = \frac{qax}{\varepsilon_s}$$

$$\rightarrow \vec{\varepsilon}(x) = \frac{qax^2}{\quad} + C$$

$$\Rightarrow \varepsilon(x) = \frac{}{2\varepsilon_s} + C$$

结合边界条件：

$$\vec{\varepsilon}\left(\pm\frac{W}{2}\right) = \frac{qa(4x^2 - W^2)}{8\varepsilon_s}$$

c. 电势分布：设P侧中性区得电势为零；

$$\psi(x) = \frac{qa}{2\varepsilon_s}\left(-\frac{x^3}{3} + \frac{W^2}{4}x + \frac{W^3}{12}\right)$$

d. 内建电势差：

$$\psi_0 = \psi\left(\frac{W}{2}\right) - \psi\left(-\frac{W}{2}\right) = \frac{qaW^3}{12\varepsilon_s}$$

另一种方法：以费米势为电势零点：

$$\begin{cases} \psi_n = V_T \ln\dfrac{n}{n_i} = V_T \ln\dfrac{aW}{2n_i} \\ \psi_p = -V_T \ln\dfrac{p}{n_i} = -V_T \ln\dfrac{aW}{2n_i} \end{cases}$$

$$\psi_0 = \psi_n - \psi_p = 2V_T \ln\frac{aW}{2n_i}$$

e. 空间电荷区宽度：

$$W = \left(\frac{12\varepsilon_s\psi_0}{qa}\right)^{\frac{1}{3}}$$

f. 例：

已知：Si线性缓变P-N结，$T = 300K, a = 10^{20}cm^{-4}, n_i = 1.5\times10^{10}cm^{-3}$；

$$\psi_0 = 2V_T \ln\frac{aW}{2n_i} = \frac{qaW^3}{12\varepsilon_s}$$

$$\Rightarrow 2\times0.026eV \ln\frac{10^{20}cm^{-4}W}{2\times1.5\times10^{10}cm^{-3}} = \frac{1.6\times10^{-19}C\times10^{20}cm^{-4}W^3}{12\times11.8\times8.85\times10^{-12}C^2/(N\cdot m^2)}$$

$$\Rightarrow \begin{cases} W \approx 0.7976\mu m \\ \psi_0 \approx 0.889V \end{cases}$$

$$\overrightarrow{\varepsilon_m} = \frac{qaW^2}{8\varepsilon_s} = \frac{3\psi_0}{2W} = \frac{3\times0.889V}{2\times0.7976\times10^{-6}m} \approx 1.672\times10^6 V/m$$