第一课：绪论

Tuesday, 10 September, 2019 10:09

1. 推荐书目：《上帝掷骰子吗？量子力学史话》；
2. 黑体：能吸收射到其上的全部辐射的物体，这种物体就称为绝对黑体，简称黑体；
3. 黑体辐射：由黑体(空腔小孔)发出的辐射称为黑体辐射；
    a. 黑体辐射只与黑体的绝对温度T有关，而与黑体的形状和材料无关；
4. 维恩(Wien)公式于短波部分实验符合，瑞利金斯公式符合长波部分；
5. 普朗克(Planck)辐射定律：
    a. 原子的性能和谐振子一样，以给定的频率ν振荡；
    b. 黑体只能以$\mathrm{E}=\hbar\nu$为能量单位不连续的发射和吸收能量，而不是像经典理论所要求的那样可以连续的发射和吸收辐射能量；
    c. $\rho_\nu d\nu=\frac{8\pi\hbar\nu^3}{c^3}\cdot\frac{1}{e^{\frac{\hbar\nu}{k_BT}}-1}d\nu$；
    d. 普朗克常量$\hbar$：$\hbar=6.62606896(33)\times10^{-34}\mathrm{J}\cdot\mathrm{s}$；
6. 爱因斯坦光电效应：$\frac{1}{2}m_e\nu_m^2=\hbar\nu-W_0$，其中$m_e$是电子的质量，$\mathrm{v_m}$是电子脱出金属表面后的速度，$\mathrm{W_0}$是金属对电子的束缚能；
    a. 电子的能量只与光的频率有关，与光强无关，光强只决定电子的数目多少；
7. 康普顿效应：高频的X射线被轻元素中的电子散射后，波长随散射角的增加而增大；
8. 德布罗意公式：
    a. $E=\hbar\nu=\hbar\omega$；
    b. $p=\frac{\hbar}{\lambda}n=\hbar k$；
9. 德布罗意波：$\Psi=\mathrm{A}e^{\frac{\mathrm{i}}{\hbar}(\mathrm{p}\cdot\mathrm{r}-\mathrm{E}t)}$，波长：$\lambda=\frac{\hbar}{\mathrm{p}}=\frac{\hbar}{\sqrt{2\mathrm{m}\mathrm{E}}}$；
10. 美国国家地理纪录片《超乎想象的宇宙》；
11. 波-粒二象性的量子力学解释：两种状态同时存在但不同时出现；
12. 波-粒特性决定的条件：测量方法；
13. 波函数的统计解释：
    a. 电子的波动性是许多电子在同一个实验(或一个电子在许多次实验中)中的统计结果；
    b. 设粒子对应的波由函数$\Psi(\vec{r},t)$描述，那么其称为波函数(一般为复函数)，则其强度为：$|\Psi(\vec{\mathrm{r}},\mathrm{t})|^2=\Psi^*(\vec{\mathrm{r}},\mathrm{t})\cdot\Psi(\vec{\mathrm{r}},\mathrm{t})$；
    c. 波函数在空间中某一点的强度和该点找到粒子的概率成正比，描述粒子的波什概率波；
    d. 经典波与概率波的区别：

经典波	概率波
振动状态	不代表物理量
波强：能流密度	波强：概率密度
无归一化	存在归一化
能量取决于波强绝对值$\lvert k\Phi(\vec{\mathrm{r}},\mathrm{t})\rvert^2;\lvert\Phi(\vec{\mathrm{r}},\mathrm{t})\rvert^2$	概率取决于波强比例$\lvert k\psi(\vec{\mathrm{r}},\mathrm{t})\rvert^2$

$= k^2$	$\left\|\frac{k\Psi(\vec{r}_1,t)}{k\Psi(\vec{r}_2,t)}\right\| = \left\|\frac{\Psi(\vec{r}_1,t)}{\Psi(\vec{r}_2,t)}\right\|^2$
描述两种不同状态	描述同一种状态

e. 波函数的性质：
   i. 连续；
   ii. 单值；
   iii. 有限；
f. 波函数是量子力学理论的基本假设：微观体系的状态被波函数完全描述，从这个波函数就可以得出体系的所有性质；

14. 课堂小结：
   a. 微观粒子的状态是量子态：粒子和波两种状态的对立统一；
   b. 测量决定微观粒子的特性；
   c. 描写粒子的波是概率波；
   d. 概率波与经典波存在区别；
   e. 波函数的性质：连续、单值、有限；
   f. 量子力学的基本假设；
15. 完。
16. 作业：
   a. 量子力学的五个基本假设；
   b. 波函数统计解的定义；
   c. 光电效应的基本实验现象及爱因斯坦对光电效应的解释；

第二课：波函数ψ和S方程

Tuesday, September 17, 2019 6:18 PM

1. 约化普朗克常量：$\hbar = \frac{h}{2\pi}$;
2. 薛定谔方程：
   a. 能量：$E = \frac{1}{2}mv^2 + v(x) = \frac{1}{2} \cdot \frac{P^2}{m} + v(x)$;
   b. 德布罗意波：$\psi = A \cdot \cos\left[\vec{k} \cdot \vec{r} - \omega t\right] = A \cdot e^{\left[i(\vec{k} \cdot \vec{r} - \omega t)\right]}$;
   c. 由：
      i. $\omega = 2\pi r$;
      ii. $k = \frac{2\pi}{\lambda}$;
      iii. $E = h\nu$;
      iv. $P = \frac{h}{\lambda}$;
   d. 得到物质的德布罗意波：$\psi = A \cdot e^{\frac{i}{\hbar} \cdot (\vec{p} \cdot \vec{r} - Et)}$;
   e. 两边对时间微分：$\frac{\partial \psi}{\partial t} = -\frac{iE}{\hbar} \cdot \psi \Rightarrow i\hbar \cdot \frac{\partial \psi}{\partial t} = E\psi$;
   f. $\frac{\partial \psi}{\partial x} = \frac{\partial}{\partial x}\left(A \cdot e^{\frac{i}{\hbar}(p_x x + p_y y + p_z z - Et)}\right) = \frac{ip_x}{\hbar} \cdot \psi \Rightarrow \frac{\partial^2 \psi}{\partial x^2} = \left(\frac{ip_x}{\hbar}\right)^2 \cdot \psi = -\frac{p_x^2}{\hbar^2} \cdot \psi$;
   g. $\Rightarrow \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = -\frac{1}{\hbar^2} \cdot \left(p_x^2 + p_y^2 + p_z^2\right)\psi$;
   h. $\Rightarrow \nabla^2 \psi = -\frac{1}{\hbar^2} p^2 \psi \Rightarrow -\frac{\hbar^2}{2m} \nabla^2 \psi = \frac{p^2}{2m} \cdot \psi$
   i. $-\frac{\hbar^2}{2m} \nabla^2 \psi + \mathrm{U(r)}\psi = \mathrm{E}\psi$;
3.

第三课：Schrodinger方程

Tuesday, September 24, 2019 7:36 PM

1. 哥本哈根解释：
   a. 概率波$|\psi|^2$；
   b. 不确定原理($p\times q \neq q\times p$)；
   c. 互补原理(粒子/波⇒量子态)
2. 波函数假设：S方程⇒ ψ；
3. 算符：$\hat{F}\cdot\psi = f\cdot\psi$；
4. 薛定谔方程：$-\frac{\hbar^2}{2m}\nabla^2\psi + \mu\psi = \frac{i\hbar\partial\psi}{\partial t}$
   a. 记作：$\hat{H}\psi = \mathrm{E}\psi = \frac{\mathrm{i}\hbar\,\partial\psi}{\partial \mathrm{t}}$ (其中$\hat{\mathrm{H}}$为哈密顿算符);
5. 定态薛定谔方程：
   a. 当势能μ与时间t无关时，势能记作$\mu(\vec{r})$，波函数为$\psi(\vec{r},t) = \psi(r)\cdot\psi(t)$；
   b. $\Rightarrow \psi(t)\left[-\frac{\hbar^2}{2m}\nabla^2\psi(r)\right] + \mu(r)\cdot\psi(r)\psi(t) = \psi(r)\cdot\frac{i\hbar\psi(t)}{\partial(t)}$;
   c. 两边同除以$\psi(\mathrm{r})\cdot\psi(\mathrm{t})$ $\Rightarrow \frac{1}{\psi(\mathrm{r})}\cdot\left(-\frac{\hbar^2}{2\mathrm{m}}\nabla^2\psi(\mathrm{r})\right) + \mu(\mathrm{r}) = \frac{1}{\psi(\mathrm{t})}\cdot\frac{\mathrm{i}\hbar^2\,\partial\psi(\mathrm{t})}{\partial \mathrm{t}} = \mathrm{E}$;
   d. 得到方程组：
      i. $\frac{i\hbar\partial\psi(t)}{\partial t} = E\cdot\psi(t)$;
      ii. $-\frac{\hbar^2}{2m}\nabla^2\psi(r) + \mu(r)\cdot\psi(r) = E\psi(r)$ (定态薛定谔方程，E确定);
   e. $\because \psi(t) = e^{\frac{E}{i\hbar}t} = e^{-\frac{iE}{\hbar}t}$;
   f. 求解得：$\psi(r,t) = \psi(r)\cdot e^{-\frac{iE}{\hbar}t}$
6. 定态的性质：
   a. 粒子在空间的几率密度($|\psi|^2 = \psi_n * \psi_n = \omega_n(\vec{r},t)$)与时间无关；
   b. 几率流密度($\vec{J_n}(\vec{r},t) = \vec{J}_n(\vec{r})$)与时间按无关；
   c. 任何不显含t的力学量平均值与t无关；
7. 一维无限深势阱：
   a. 薛定谔方程的一维形式：$-\frac{\hbar^2}{2m}\nabla^2\psi(x) + \mu(x)\cdot\psi(x) = E\psi(x)$；
   b. 一维无限深势阱：
      i. $V(x) = \begin{cases} 0, & |x| \le a, \\ \infty, & |\mathrm{x}| > \mathrm{a} \end{cases}$;
   c. $\because -\frac{\hbar^2}{2\mu}\cdot\frac{d^2}{dx^2}\psi(x) + V(x)\psi(x) = E\psi(x) \Rightarrow \frac{d^2}{dx^2}\psi(x) - \frac{2\mu}{\hbar^2}[V(x)-E]\psi(x) = 0$;
   d. 各势区的S方程：
      i. $\frac{d^2}{dx^2}\psi^I(x) - \frac{2\mu}{\hbar^2}(V-E)\psi^I(x) = 0,\ x \le -a$;
      ii. $\frac{d^2}{dx^2}\psi^{II}(x) + \frac{2\mu}{\hbar^2}E\psi^{II}(x) = 0,\ -a < x < a$ (记$\frac{2\mu}{\hbar^2}E = \alpha^2$);
      iii. $\frac{d^2}{dx^2}\psi^{III}(x) - \frac{2\mu}{\hbar^2}(V-E)\psi^{III}(x) = 0,\ x \le -a$ (记$\frac{2\mu}{\hbar^2}(V-E) = \beta^2$);
   e. 简记作：
      $\frac{d^2}{dx^2}\psi^I - \beta^2\psi^I = 0$
      $\frac{d^2}{dx^2}\psi^{II} + \alpha^2\psi^{II} = 0$;
      $\frac{d^2}{dx^2}\psi^{III} - \beta^2\psi^{III} = 0$
   f. 因为波函数具有周期性；
   g. $\alpha^2 = \left(\frac{n\pi}{a}\right)^2 = \frac{2\mu}{\hbar^2}E \Rightarrow E = \frac{\hbar^2}{2\mu}\cdot\left(\frac{n\pi}{a}\right)^2 = \frac{\hbar^2 n^2\pi^2}{2\mu a^2}$;
   h. $\begin{vmatrix} A\cdot\sin(-\alpha a+\delta) = 0 & \Rightarrow & A\cdot\sin(-\alpha a)\cos\delta + A\cdot\cos(\alpha a)\sin\delta = 0 \\ A\cdot\sin(\alpha a+\delta) = 0 & \Rightarrow & A\cdot\sin(\alpha a)\cos\delta + A\cdot\cos(\alpha a)\sin\delta = 0 \end{vmatrix}$;
   i. $\Rightarrow \begin{vmatrix} \cos(\alpha a)\sin\delta = 0 & \Rightarrow & \sin\delta = 0\ or\ \cos\alpha a = 0 \\ \sin(\alpha a)\cos\delta = 0 & \Rightarrow & \cos\delta = 0\ or\ \sin\alpha a = 0 \end{vmatrix}$;
   j. 分析其中两种情况：
      i. $\sin\delta = 0 \Rightarrow \delta = 0$;
         1) $\cos\delta = 1 \Rightarrow \sin\alpha a = 0$;
         2) $\because \alpha^2 = \frac{2\mu}{\hbar^2}E$;
         3) $\alpha a = n\pi \Rightarrow \alpha = \frac{n\pi}{a}\ (n = 0, \pm1, \pm2, \dots)$;
         4) $\Rightarrow E = \frac{\hbar^2}{2\mu}\alpha^2 = \frac{\hbar^2}{2\mu}\left(\frac{n\pi}{a}\right)^2 = \frac{n^2\pi^2\hbar^2}{2\mu a^2} = E_n$;
         5) $\psi_n^{II} = A\cdot\sin\alpha x = A\cdot\sin(\frac{n\pi}{a}x)$;
      ii. $\cos\delta = 0 \Rightarrow \delta = \frac{\pi}{?}$;

1) $\sin\delta = 1 \Rightarrow \cos\alpha a = 0$;

2) $E_n = \frac{\hbar^2}{2\mu}\alpha^2 = \frac{\hbar^2}{2\mu}\left(\frac{\left(n+\frac{1}{2}\right)\pi}{a}\right)^2 = \frac{(2n+1)^2\pi^2\hbar^2}{8\mu a^2}$;

3) $\psi_n = \begin{cases} \psi^{\mathrm{I}} = \psi^{\mathrm{III}} = 0 \\ \psi_{\mathrm{n}}^{\mathrm{II}} = \mathrm{A}\sin\left(\alpha a + \frac{\pi}{2}\right) = \mathrm{A}\cos\alpha a = \mathrm{A}\cos\frac{n+\frac{1}{2}}{a}\pi x = \mathrm{A}\cos\frac{2n+1}{2a}\pi x \end{cases} (\mathrm{n} = 0,1,2,\ldots)$;

k. 综上两种情况；

i. $E_m = \frac{m^2\pi^2\hbar^2}{8\mu a^2}$;

ii. $\psi_m = \begin{cases} \begin{cases} \psi^I = \psi^{III} = 0 \\ \psi^{II} = A\sin\frac{m\pi}{2a}x \end{cases} (m = 2n, n \in Z) \\ \begin{cases} \psi^I = \psi^{III} = 0 \\ \psi^{II} = A\cos\frac{m\pi}{2a}x \end{cases} (m = 2n+1, n \in Z) \end{cases}$;

l. 导出归一化系数A：$\int|\psi(x)|^2 dx = 1$；

m. 将定态薛定谔方程还原为薛定谔方程：$\psi(x,t) = \psi(x)\cdot\psi(t)$,其中$\psi(t) = e^{-\frac{iEnt}{\hbar}}$;

第四课：算符

Tuesday, October 8, 2019 6:57 PM

1. 动量算符:$\hat{\vec{p}} = -\mathrm{i}\hbar(\frac{\partial}{\partial \mathrm{x}}\hat{\mathrm{e}}_{\mathrm{x}} + \frac{\partial}{\partial \mathrm{y}}\hat{\mathrm{e}}_{\mathrm{y}} + \frac{\partial}{\partial \mathrm{z}}\hat{\mathrm{e}}_{\mathrm{z}})$;
2. 能量算符:$\widehat{H} = -\frac{\hbar}{2\mu}\nabla^2 + U(x,y,z)$;
3. 角动量算符:$\hat{L} = -i\hbar(\hat{r}\times\nabla)$;
   a. $\hat{L}^2 Y_{lm} = l\times(l+1)\hbar Y_{lm}$
   b. $\widehat{L_z} Y_{lm} = m\hbar Y_{lm}$
4. 厄米算符:$\exists \hat{F} \Rightarrow \int \psi^* \hat{\mathrm{F}} \phi \mathrm{d}\tau = \int \left(\hat{\mathrm{F}}\psi\right)^* \phi \mathrm{d}\tau \text{ or } \hat{\mathrm{F}} = \hat{\mathrm{F}}^+$;

第五课：球坐标系

Tuesday, October 8, 2019 7:03 PM

1. 角动量$L=\vec{r}\times\vec{p}$;
2. $\begin{cases} L=\vec{r}\times i\hbar\nabla=i\hbar\vec{r}\times\nabla \\ \vec{\mathrm{r}}=\mathrm{xi}+\mathrm{yj}+\mathrm{zk} \\ \nabla=\frac{\partial}{\partial \mathrm{x}}\mathrm{i}+\frac{\partial}{\partial \mathrm{y}}\mathrm{j}+\frac{\partial}{\partial \mathrm{z}}\mathrm{k} \end{cases}$;
3. $\Rightarrow L=i\hbar\vec{r}\times\nabla=i\hbar\begin{vmatrix} i & j & k \\ x & y & z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{vmatrix}=i\hbar\left[\left(y\cdot\frac{\partial}{\partial z}-z\cdot\frac{\partial}{\partial y}\right)i+\left(z\cdot\frac{\partial}{\partial x}-x\cdot\frac{\partial}{\partial z}\right)j+\left(x\cdot\frac{\partial}{\partial y}-y\cdot\frac{\partial}{\partial x}\right)k\right]$;
4. 令:$L=L_x i+L_y j+L_z k$;
5. $\begin{cases} x=r\cdot\sin\theta\cdot\cos\phi \\ y=r\cdot\sin\theta\cdot\sin\phi \\ z=r\cdot\cos\theta \end{cases}\Rightarrow\begin{cases} r^2=x^2+y^2+z^2 \\ \cos\theta=\frac{z}{r} \\ \frac{y}{x}=tg\ \phi \end{cases}$;
6. $\frac{\partial}{\partial x}=\frac{\partial}{\partial r}\cdot\frac{\partial r}{\partial x}+\frac{\partial}{\partial\theta}\cdot\frac{\partial\theta}{\partial x}+\frac{\partial}{\partial\phi}\cdot\frac{\partial\phi}{\partial x}$;
7. $\because r^2=x^2+y^2+z^2\Rightarrow\frac{dr}{dx}\cdot 2r=2x\Rightarrow\frac{dr}{dx}=\frac{x}{r}$;
8. $\because x=r\cdot\sin\theta\cdot\cos\phi\Rightarrow\frac{dr}{dx}=\sin\theta\cdot\cos\phi$;
9. $\because r\cdot\cos\theta=z\Rightarrow\frac{dr}{dx}\cdot\cos\theta+r\cdot(-\sin\theta)\cdot\frac{d\theta}{dx}=0\Rightarrow\frac{d\theta}{dx}=\frac{\cos\phi\cdot\cos\theta}{r}$;
10. $\because y=x\cdot tg\ \phi\Rightarrow\theta=tg\ \phi+x\cdot\sec\phi\cdot\frac{d\phi}{dx}\Rightarrow\frac{d\phi}{dx}=-\frac{\sin\phi}{\cos\phi}\cdot\frac{1}{x}\cdot\cos\phi\cdot\cos\phi=-\frac{\sin\phi\cdot\cos\phi}{r\cdot\sin\theta\cdot\cos\phi}=-\frac{1}{r}\cdot\frac{\sin\phi}{\sin\theta}$;
11. $\Rightarrow\frac{\partial}{\partial x}=\sin\theta\cdot\cos\phi\cdot\frac{\partial}{\partial r}+\frac{\cos\theta\cdot\cos\phi}{r}\cdot\frac{\partial}{\partial\theta}+\left(-\frac{1}{r}\right)\cdot\frac{sin\phi}{sin\theta}\cdot\frac{\partial}{\partial\phi}$;
12. 同理：$\begin{cases} \frac{\partial}{\partial x}=\sin\theta\cdot\cos\phi\cdot\frac{\partial}{\partial r}+\frac{\cos\theta\cdot\cos\phi}{r}\cdot\frac{\partial}{\partial\theta}+\left(-\frac{1}{r}\right)\cdot\frac{sin\phi}{sin\theta}\cdot\frac{\partial}{\partial\phi} \\ \frac{\partial}{\partial y}=sin\theta\cdot sin\phi\cdot\frac{\partial}{\partial r}+\frac{1}{r}cos\theta\cdot sin\phi\cdot\frac{\partial}{\partial\theta}+\frac{1}{r}\frac{cos\phi}{sin\theta}\cdot\frac{\partial}{\partial\phi} \\ \frac{\partial}{\partial z}=cos\theta\cdot\frac{\partial}{\partial r}+\left(-\frac{1}{r}\right)sin\theta\cdot\frac{\partial}{\partial\theta}+0 \end{cases}$;
13. $\because L^2=L_x^2+L_y^2+L_z^2$;
14. $\Rightarrow L^2=-\hbar^2[\left(y\cdot\frac{\partial}{\partial z}-z\cdot\frac{\partial}{\partial y}\right)^2+\left(z\cdot\frac{\partial}{\partial x}-x\cdot\frac{\partial}{\partial z}\right)^2+\left(x\cdot\frac{\partial}{\partial y}-y\cdot\frac{\partial}{\partial x}\right)^2]$;
15. $\Rightarrow L^2=-\hbar^2\left[\frac{1}{sin\theta}\cdot\frac{\partial}{\partial\theta}\left(sin\theta\cdot\frac{\partial}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\cdot\frac{\partial^2}{\partial\phi^2}\right]$;
16. 本征方程：算符作用在波函数上等于一个确定的值乘波函数;
17. $\widehat{L_z}=-i\hbar\cdot\frac{\partial}{\partial\phi}\Rightarrow\widehat{L_z}\psi=L_z\cdot\psi\Rightarrow-i\hbar\cdot\frac{\partial}{\partial\phi}\psi=L_z\psi$($\mathrm{L_z}$为实数，可观测);
18. $\Rightarrow\frac{\partial\psi}{\partial\phi}=1=-\frac{1}{i\hbar}\cdot L_z\cdot\psi=\frac{i}{\hbar}\cdot\mathrm{L}_z\cdot\psi$;
19. $\psi=ce^{\frac{iL_z}{\hbar}\phi}$;
20. 由角动量边界条件：$\psi(\phi)=\psi(\phi+2\pi)$;
21. $ce^{\frac{iL_z}{\hbar}\phi}=ce^{\frac{iL_z}{\hbar}(\phi+2\pi)}$;
22. $\because e^{\frac{iL_z2\pi}{\hbar}}=\cos\left(\frac{L_z2\pi}{\hbar}\right)+isin\left(\frac{L_z2\pi}{\hbar}\right)=1\ and\ \frac{L_z2\pi}{\hbar}=2\pi m\Rightarrow L_z=m\hbar$;
23. $\because\int_0^{2\pi}|\psi|^2d\psi=1\Rightarrow\int_0^{2\pi}c^2\left|e^{im\phi}\right|^2d\psi=1\Rightarrow c^2\int_0^{2\pi}1d\psi=1\Rightarrow c=\sqrt{\frac{1}{2\pi}}$;
24. $\begin{cases} L_z=m\hbar \\ \psi_m(\phi)=\frac{1}{\sqrt{2\pi}}e^{im\phi} \end{cases}$;
25. $\widehat{L_2}Y(\theta,\phi)=\lambda\hbar^2Y(\theta,\phi)$;
26. $-\hbar^2\left[\frac{1}{sin\theta}\cdot\frac{\partial}{\partial\theta}\left(sin\theta\cdot\frac{\partial}{\partial\theta}\right)+\frac{1}{sin^2\theta}\cdot\frac{\partial^2}{\partial\phi^2}\right]Y(\theta,\phi)=\lambda\hbar^2Y(\theta,\phi)$;

26. $-\hbar^2\left[\frac{1}{sin\theta}\cdot\frac{\partial}{\partial\theta}\left(sin\theta\cdot\frac{\partial}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\cdot\frac{\partial^2}{\partial\phi^2}\right]Y(\theta,\phi)=\lambda\hbar^2Y(\theta,\phi)$;
27. $\widehat{L_z^2}=-i\hbar\cdot\frac{\partial}{\partial\phi}\Rightarrow L_z^2=-\hbar^2\cdot\frac{\partial^2}{\partial\phi^2}$;
28. 式$-\hbar^2\left[\frac{1}{sin\theta}\cdot\frac{\partial}{\partial\theta}\left(sin\theta\cdot\frac{\partial}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\cdot\frac{\partial^2}{\partial\phi^2}\right]Y(\theta,\phi)=\lambda\hbar^2Y(\theta,\phi)$中，前半部分是关于$\theta$的函数，后半部分关于$\phi$，因此可以分离变量;
29. $\because\widehat{L_z}\phi(\phi)=m\hbar\phi(\phi)$;
30. $Y(\theta,\phi)=\theta(\theta)\cdot\phi(\phi)\Rightarrow L_z^2\phi(\phi)=m^2\hbar^2\phi(\phi)$;
31. 消掉ϕ $\Rightarrow\left[-\frac{\hbar^2}{sin\theta}\cdot\frac{\partial}{\partial\theta}\left(sin\theta\cdot\frac{\partial}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\cdot m^2\hbar^2\right]Y(\theta,\phi)=\lambda\hbar^2Y(\theta,\phi)$;
32. 解得：$\frac{1}{sin\theta}\cdot\frac{\partial}{\partial\theta}\left(sin\theta\cdot\frac{\partial}{\partial\theta}\right)+\left(\lambda-\frac{m^2}{\sin^2\theta}\right)\theta=0$;
33. $Y(\theta,\phi)=\begin{cases}(-1)^mN_{lm}\cdot P_l^m(cos\theta)e^{im\phi}, m=0\\(-1)^mY_{1-m}^*(\theta,\phi), m=-1\end{cases}, N_{lm}=\sqrt{\frac{(1-|m|)!(2l+1)}{4\pi(1+|m|)!}}$;
34. l表示角量子数，m表示磁量子数;
35. 当l确定时，m可取$0,\pm1,\pm2\ldots,\pm l$,此时简并度为2l+1;

第六课：再谈算符

Tuesday, October 15, 2019 10:40 AM

1. 算符作用在波函数上等于一个确定的值乘波函数：$\hat{F}\psi = \mathrm{f}\cdot\psi$；
2. 厄米算符的性质：
   1. 厄米算符的平均值时实数：
      $\bar{F} = \int d\tau\psi^*\hat{F}\psi = \int d\tau(\hat{F}\psi) * \psi = \int d\tau\left(\psi^*(\hat{F}\psi)\right)^* = \bar{F}^*$
      ∵ $\bar{F} = \bar{F}^*$ ⇒ F̄是实数
   2. 平均值都是实数的算符必定是厄米算符(1的逆定理)：
      $\bar{F} = \bar{F}^*$
      左侧: $\bar{F} = \int \psi^*\hat{F}\psi d\tau = \int d\tau(\bar{F}\psi)^*\psi$
      取$\psi = \psi_1 + \mathrm{c}\psi_2$代入F̄ $= \int d\tau\psi^*\hat{F}\phi = \int d\tau(\psi_1 + c\psi_2)^*\hat{F}(\psi_1 + c\psi_2)$
      $\int d\tau\psi_1^*\hat{F}\psi_1 + |c|^2\int d\tau\psi_2^*\hat{F}\psi_2 + c\int d\tau\psi_1^2\hat{F}\psi_2 + \int d\tau c\psi_2^*\hat{F}\psi_1$
      $\int d\tau(\hat{F}\psi)^*\psi = \int d\tau\left(\hat{F}(\psi_1 + c\psi_2)\right)^*(\psi_1 + c\psi_2)$
      $= \int d\tau(\hat{F}\psi_1)^*\psi_1 + \int(\hat{F}\psi_1)^*c\psi_2 + \int d\tau(\hat{F}c\psi_2)^*\psi_1 + \int d\tau(\hat{F}c\psi_2)c\psi_2$
      $\int d\tau(\hat{F}\psi_1)^*\psi_1 = \left(\int d\tau\psi_1^*(\hat{F}\psi_1)\right)^*$
      $\int d\tau(\hat{F}\psi_2)^*\psi_2 = \left[\int d\tau\psi_2^*(F\psi_2)\right]^*$
      ⇒ $\bar{F} = \bar{F}^*$
      ∵ 左式 = 右式
      ⇒ $c\left(\int d\tau\psi_1^*\hat{F}\psi_2 - \int d\tau(F\psi_1)^*\psi_2\right) = c^*\left(\int d\tau(\hat{F}\psi_2)^*\psi_1 - \int d\tau\psi_2^*\hat{F}\psi_1\right)$
      令c=1:$\int d\tau\psi_1^*\hat{F}\psi_2 - \int d\tau(F\psi_1)^*\psi_2 = \int d\tau(\hat{F}\psi_2)^*\psi_1 - \int d\tau\psi_2^*\hat{F}\psi_1$
      令c=i:$\int d\tau\psi_1^*\hat{F}\psi_2 - \int d\tau(F\psi_1)^*\psi_2 = -\left(\int d\tau(\hat{F}\psi_2)^*\psi_1 - \int d\tau\psi_2^*\hat{F}\psi_1\right)$
      $\begin{cases}\int d\tau\psi_1^*\hat{F}\psi_2 = \int d\tau(F\psi_1)^*\psi_2 \\ \int d\tau\psi_2^*\hat{F}\psi_1 = \int d\tau(F\psi_2)^*\psi_1\end{cases}$(厄米算符表达式)
   3. 本征方程：$\hat{F}\psi = \mathrm{f}\cdot\psi$；
      涨落：$\overline{|\Delta F|^2} = \overline{\left(\hat{F} - \bar{F}\right)^2} = \int\psi^*\left(\hat{F} - \bar{F}\right)\psi d\tau$，其中：F̂是厄米算符,F̄是实数,
      其平均值是实数；
      $\bar{F}^2 = \int d\tau\psi^*F^2\psi = \int d\tau\psi^*\hat{F}\hat{F}\psi d\tau = \int d\tau(\hat{F}\psi)^*(\hat{F}\psi) = \int d\tau\left|\hat{\mathrm{F}}\psi\right|^2 \geq 0$
      把F换成ΔF：
      $\overline{|\Delta F|^2} = \int\left|\hat{F} - \bar{F}\psi\right|^2 \geq 0$
      当$\overline{|\Delta F|^2} = 0$时，测量涨落最小，即$\left|\hat{F} - \bar{F}\right|\psi = 0$，$\hat{F}\psi = \bar{\mathrm{F}}\psi$是本征方程；
   4. 正交归一性：厄米算符属于不同本征值的本征函数正交；
      $\hat{F}\phi_n = F_n\phi_n, \hat{F}\phi_m = F_m\phi_m$，证明它们正交。
      取 $\hat{F}\phi_m = F_m\phi_m$的共轭$\left(\hat{F}\phi_m\right)^* = (F_m\phi_m)^* = F_m\cdot\phi_m^*$
      右乘ϕ_n并积分：$\int\left(\hat{F}\phi_m\right)^*\phi_n d\tau = F_m\cdot\int\phi_m^*\phi_n d\tau$
      由厄米算符的性质：$\int\left(\hat{F}\phi_m\right)^*\phi_n d\tau = \int\phi_m^*\hat{F}\phi_n d\tau = \int\phi_m^*F_n\phi_n d\tau = F_n\int\phi_m^*\phi_n d\tau = F_m\cdot\int\phi_m^*\phi_n d\tau$
      ∵ $\forall F_m \neq F_n \Rightarrow \int\phi_m^*\phi_n d\tau = 0 (m \neq n)$(正交性)

$\int \phi_m^* \phi_n d\tau = 1(m=n)$(归一性)

正交归一性$\int \phi_m^* \phi_n d\tau = \theta_{mn}$

5. 完备性：$\hat{F}\phi_n = \lambda_n \phi_n (n \in N^*)$

λ_n: $\lambda_1, \lambda_2, \lambda_3 \ldots$(本征值)

ϕ_n: $\phi_1, \phi_2, \phi_2 \ldots$(本征函数)

对于厄米算符$\hat{F}$:

本征函数任意$\psi(x)$可以按$\phi_n(x)$展开：$\psi(x) = \Sigma_n c_n \phi_n(x)$

左乘ϕ_m^*并积分：$\int \phi_m^* \psi(x) = \int \phi_m^* \Sigma_n c_n \phi_n d\tau$

$\Rightarrow \int \phi_m^* \psi(x) d\tau = \Sigma_n c_n \int \phi_m^* \phi_n d\tau = \Sigma_n c_n \cdot \theta_{mn} = c_m$

6. 当$\psi(x)$归一化时，系数c_n也是归一化的：

$1 = \int \psi^*(x)\psi(x)d\tau, \quad \psi(x) = \Sigma_n c_n \phi_n$

$\Rightarrow 1 = \int (\Sigma_n C_N \phi_n)^* \cdot (\Sigma_m c_m \phi_m) d\tau = \Sigma_n \Sigma_m c_n c_m \int \phi_n^* \phi_m d\tau = \Sigma_n \Sigma_m c_n c_m \theta_{mn}$

$= \Sigma_n |c_n|^2 = 1$

7. 力学量的平均值：$\bar{F} = \int \psi^* \hat{F} \psi d\tau$

一维情况下：$\bar{F} = \int \psi^*(x)\hat{F}\psi(x)dx = \int (\Sigma_n c_n \phi_n)^* \hat{F} (\Sigma_m c_m \phi_m) dx$

$\because \hat{F}\phi_n = \lambda_n \phi_n$

$\Rightarrow \int (\Sigma_n c_n \phi_n)^* \hat{F} (\Sigma_m c_m \phi_m) dx = \int (\Sigma_n c_n \phi_n)^* \lambda_m \Sigma_m c_m \phi_m dx$

$= \Sigma_n \Sigma_m c_n c_m \lambda_m \int \phi_n^* \phi_m dx = \Sigma_n |c_n|^2 \cdot \lambda_n$

$\bar{F} = \Sigma_n |c_n|^2 \lambda_n$

其中$|c_n|^2$是本征值出现的几率；

当$|c_n|^2$是归一化的时候，λ_n是本征值；

当$|c_n|^2$非归一化时，$\frac{c_1^2}{c_1^2 + c_2^2 + \cdots + c_n^2} = \lambda_n$

第七课：算符性质

Tuesday, October 22, 2019 6:39 PM

1. 两个算符的关系：$\begin{cases}\text{对易}\\ \text{不对易}\end{cases}$
   a. 对易：$[\hat{o},\hat{\mu}]\equiv\hat{o}\hat{\mu}-\hat{\mu}\hat{o}=0\Rightarrow\hat{o}\hat{\mu}=\hat{\mu}\hat{o}$；
      1. 两个算符同时有确定值⇔ ψ是共同本征函数；
      2. 两算符有一组共同的完备的本征函数系⇔两算符对易；
      3. 一组力学量算符具有共同完备的本征函数系⇔它们两两对易；
   b. 不对易：设$\hat{F}\hat{G}-\hat{G}\hat{F}=ik=[\hat{F},\hat{G}]$称为对易关系；
2. 示例：证明测不准原理。

   引入$I(\xi)=\int\left|\xi\widehat{\Delta F}\psi-i\widehat{\Delta G}\psi\right|^2d\Omega\geq0$

   $$=\int\left(\xi\widehat{\Delta F}\psi-i\widehat{\Delta G}\psi\right)^*\left(\xi\widehat{\Delta F}\psi-i\widehat{\Delta G}\psi\right)d\Omega$$

   $$=\int\left(\xi^2(\Delta F\psi)^*\widehat{\Delta F}\psi-i\xi(\Delta\mathrm{F}\psi)^*\widehat{\Delta G}\psi\text{-}i\xi(\Delta G\psi)^*\Delta F\psi+\left(\widehat{\Delta G}\psi\right)^*\Delta G\psi\right)d\Omega$$

   $\because\Delta F=\hat{F}-\bar{F}$

   $(\Delta F)^+=\left(\hat{F}-\bar{F}\right)^+$

   $=\hat{F}^+-\bar{F}^+$

   $=\Delta F$

   $\Rightarrow\Delta F$是厄米算符

   $$=\xi^2\int\psi^*\widehat{\Delta F}^2\psi d\tau-i\xi\int\psi^*\left(\widehat{\Delta F}\Delta G-\Delta G\Delta F\right)\psi d\tau+\int\psi^*(\Delta G)^2\psi d\tau$$

   $\widehat{\Delta F}\Delta G-\Delta G\Delta F$

   $\equiv[\Delta F,\Delta G]$

   $=\left[\hat{F}-\bar{F},\hat{G}-\bar{G}\right]$

   $=\left[\hat{F},\hat{G}\right]-\left[\hat{F},\bar{G}\right]-\left[\bar{F},\hat{G}\right]+[\bar{F},\bar{G}]$

   $=\left[\hat{G},\hat{G}\right]$

   $=ik$

   $=\xi^2\overline{(\Delta F)^2}+\xi\bar{k}+\left(\widehat{\Delta\mathrm{G}}\right)^2\geq0$ (以ξ为变量的二次函数)

   $\because b^2-4ac\leq0$

   $\Rightarrow\bar{k}^2-4\overline{(\Delta F)^2}\cdot\overline{(\Delta\mathrm{G})^2}\leq0$

   $\Rightarrow\overline{(\Delta F)^2}\cdot\overline{(\Delta G)^2}\geq\dfrac{\bar{k}^2}{4}$

   当ΔF → Δx, ΔG → Δp:

   $[x,p_x]=i\hbar$

   $\Rightarrow\overline{(\Delta x)^2}\cdot\overline{(\Delta p)^2}\geq\dfrac{\hbar^2}{4}$

   证得：$\Delta x\cdot\Delta p\geq\frac{\hbar}{2}$，即测不准原理；
3. 示例：
   a. 证明：$\left[\widehat{L_x},\widehat{L_y}\right]=i\hbar L_z$

   $\left[y\widehat{P_z}-z\widehat{P_y},z\widehat{P_x}-x\widehat{P_z}\right]$

   $=\left[y\widehat{P_z},z\widehat{P_x}\right]-\left[y\widehat{P_z},x\widehat{P_z}\right]-\left[z\widehat{P_y},z\widehat{P_x}\right]+\left[z\widehat{P_y},x\widehat{P_z}\right]$

   $=\left[y\widehat{P_z},z\widehat{P_x}\right]+\left[z\widehat{P_y},x\widehat{P_z}\right]$

   $=y\left[P_z,z\widehat{P_x}\right]+\left[y,z\widehat{P_x}\right]\widehat{P_z}+z\left[\widehat{P_y},x\widehat{P_z}\right]+\left[z,x\widehat{P_z}\right]\widehat{P_y}$

   $=y\left[\widehat{P_z},z\widehat{P_x}\right]+\left[z,x\widehat{P_z}\right]\widehat{P_y}$

   $=yz\left[\widehat{P_z},\widehat{P_x}\right]+y\left[\widehat{P_z},z\right]\widehat{P_x}+x\left[z,\widehat{P_z}\right]\widehat{P_y}+[z,x]\widehat{P_y}\widehat{P_z}$

   $\because[P_z,X_z]=-i\hbar$

   $\Rightarrow$ 原式 $=y(-i\hbar)\widehat{P_x}+x\cdot i\hbar\widehat{P_y}$

   $=i\hbar\left(x\widehat{P_y}-y\widehat{P_x}\right)$

$= i\hbar\widehat{L_z}$

b. 证明：L_z的本征态Y_{lm}下，$<L_x>=<L_y>=0$

$[L_y, L_z] = i\hbar L_x \Rightarrow \widehat{L_x} = \frac{L_y \cdot L_z - L_z \cdot L_y}{i\hbar}$

$\because L_z Y_{lm} = m\hbar Y_{lm}$

$<L_x>=\frac{1}{i\hbar}\int Y_{lm}^*(L_y \cdot L_z - L_z \cdot L_y) \cdot Y_{lm} d\Omega$

$= \frac{1}{i\hbar}\int (Y_{lm}^* L_y L_z Y_{lm} - Y_{lm}^* L_z L_y Y_{lm}) d\Omega$

$= \frac{1}{i\hbar}\int (Y_{lm}^* L_y (m\hbar) Y_{lm} - (L_z Y_{lm})^* L_y Y_{lm}) d\Omega$

$= \frac{1}{i\hbar}\int (Y_{lm}^* L_y (m\hbar) Y_{lm} - (m\hbar) Y_{lm}^* L_y Y_{lm}) d\Omega$

$= 0$

c. 用测不准原理证明L_z在Y_{lm}下，$<L_x>=<L_y>=0$

$\because \overline{(\Delta L_y)^2} \cdot \overline{(\Delta L_z)^2} \geq \frac{\hbar^2}{4}\overline{L_x}^2$，　　L_z在Y_{lm}下$\overline{\Delta L_z}^2 = 0$

$\Rightarrow \frac{\hbar^2}{4}\overline{L_x}^2 \leq 0$

$\Rightarrow \overline{L_x}^2 \leq 0$

$\Rightarrow \overline{L_x}^2 = 0$

$\Rightarrow \overline{L_x} = 0$

$<L_x>=0$

同理$<L_y>=0$

d. 在$\hat{L}^2$和$\widehat{L_z}$共同本征态Y_{lm}下，求$\overline{(\Delta L_z)^2}$和$\overline{(\Delta L_y)^2}$

$\overline{\Delta F}^2$

$= \overline{(\hat{F} - \bar{F})^2}$

$= \overline{\hat{F}^2 - 2\hat{F} \cdot \bar{F} + \bar{F}^2}$

$= \overline{F^2} - 2\bar{\hat{F}} \cdot \bar{F} + \bar{F}^2$

$= \overline{F^2} - \bar{F}^2$

$\overline{\Delta L_x}^2 = \overline{L_x^2} - \overline{L_x}^2$

$\because \overline{L_x} = \overline{L_y} = 0$

$\Rightarrow \overline{\Delta L_x}^2 = \overline{L_x^2} = \int Y_{lm}^* L_x^2 Y_{lm} d\Omega$

$i\hbar L_x = [L_y, L_z]$

$= L_y \cdot L_z - L_z \cdot L_y$

$\Rightarrow i\hbar L_x \cdot L_x = L_y L_z \cdot L_x - L_z L_y \cdot L_x$

$\because L_z Y_{lm} = m\hbar Y_{lm}$

$\Rightarrow \quad = \frac{(L_y(L_x L_z + i\hbar L_y) - L_z L_y L_x)}{i\hbar}$

$\because [L_z, L_z] = i\hbar L_y$

$\Rightarrow \quad = \frac{1}{i\hbar}\int Y_{lm}^*(L_y L_x L_z + i\hbar L_y^2 - L_z L_y L_x) Y_{lm} d\Omega$

$= \frac{1}{i\hbar}\int Y_{lm}^*(m\hbar L_y L_x + i\hbar L_y^2 - m\hbar L_y L_x) Y_{lm} d\Omega$

$= \frac{1}{i\hbar}\int Y_{lm}^*(i\hbar L_y^2) Y_{lm} d\Omega$

$= \overline{L_y^2}$

$L_x^2 + L_y^2 + L_z^2 = L^2$

$= + L_x^2 + L_y^2 = L^2 - L_z^2$

在Y_{lm}下取平均:

$\int Y_{lm}^*(L_x^* + L_y^*) Y_{lm} d\Omega = \int Y_{lm}^*(L^2 - L_z^2) Y_{lm} d\Omega$

$\overline{L_x^2} + \overline{L_y^2} = l(l+1)\hbar^2 - (m\hbar)^2$

$\overline{L_x^2} = \overline{L_y^2} = \frac{[l(l+1) - m^2]\hbar^2}{2}$

第八课：需要了解

Tuesday, October 22, 2019 8:10 PM

1. 线性谐振子的势能函数；
2. 为什么研究线性谐振子；
3. 势垒贯穿(隧道效应)：粒子能够穿透比它动能更高的势垒的现象，它是粒子具有波动性的生动表现；